An optimization model for line planning and timetabling in automated urban metro subway networks. A case study.

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joint work with V. Blanco (UGR), E. Conde (US) and J. Puerto (US)

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Motivation of the Project

R&D Company interested on implementing automatic subway networks in Europe.

Contract: 1853/0257 (Société Metrolab®, Service Contrôle de Gestion), 2014.

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Submitted to *Omega*.

Outline

1. Problem Description
2. A mathematical programming model
3. Algorithm
4. Case Study and Computational Experiments
5. Extensions

Problem Description

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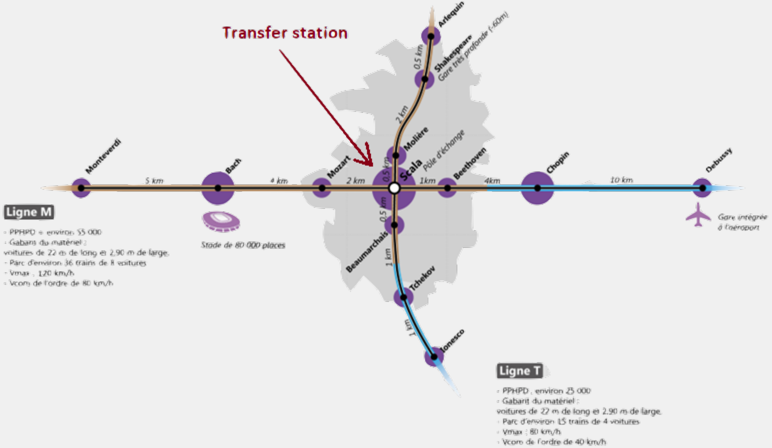

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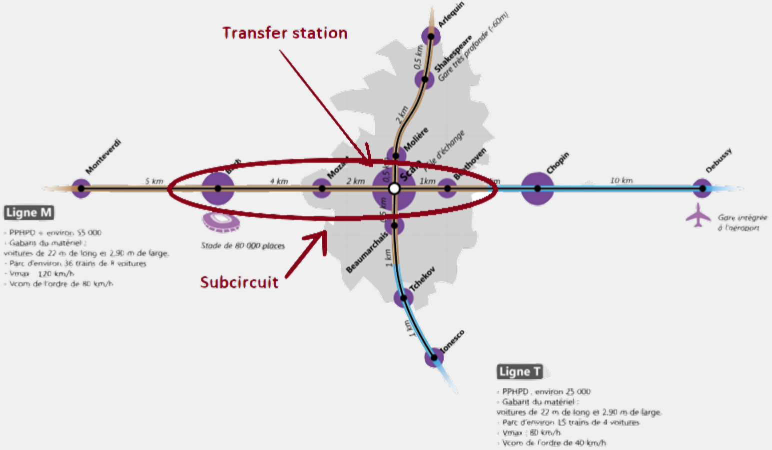

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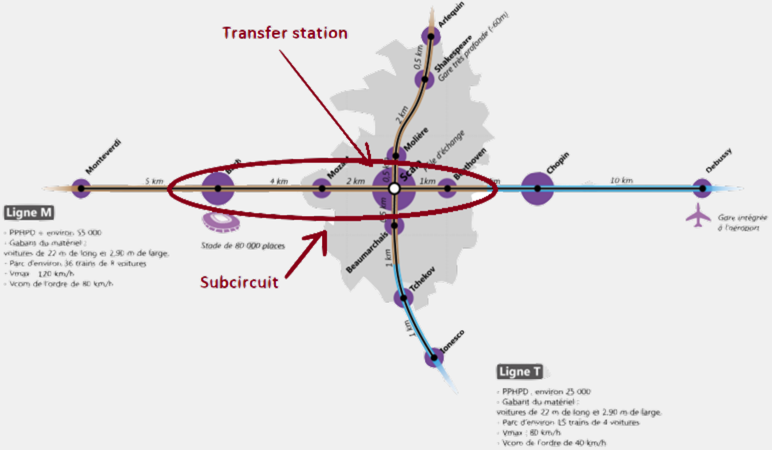

Goal: Optimizing the operations of the subway network (a pilot experience to evaluate the costs and needs in automatic subway networks).

Public Transportation Planning

1. **network design**, where the stations, links and routes of the lines are established,
2. **line planning**, specifying the frequency and the capacity of the vehicles used in each line.
3. **timetabling**, defining the arrival/departure times and
4. **scheduling**, in which vehicles and/or crews are planned.

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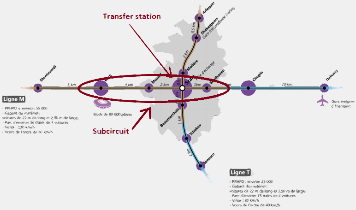

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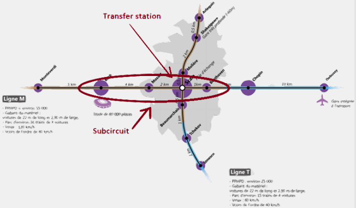

A very complex problem: multiobjective, multilevel, stochastic, combinatorial, ...

Some simplifications must be assumed in order to obtain operational solutions. The usefulness of the model will be conditioned by these assumptions.

A *what-if* tool to make rational decisions.

Problem Description

Input Data

- Structure of the network (no. of lines, stations, distances, speed, stations of the short-turns,...).
- Possible Capacities for the trains (based on the carriages).
- Safe times between trains.
- Passengers flow between each O-D (can be assumed uniformly distributed in time windows of the planning horizon).

Problem Description

Goal: Optimizing the operations of the subway network.

- Minimize the operative costs (no. of rounds, capacities,..).
- Minimize the no. of passengers exceeding effective capacities.
- Maximize the profit (by passengers use).

Problem Description

Decisions

- Number of rounds (complete lines and short-turns) over the same line to be planned in the time horizon.
- Capacities (among the available) for each of the trains in a route.
- Timetables for each of the lines operating in the system.

A mathematical programming model

1. The whole planning is partitioned into different time windows (with homogeneous demand): peak, off-peak, etc.. hours

A mathematical programming model

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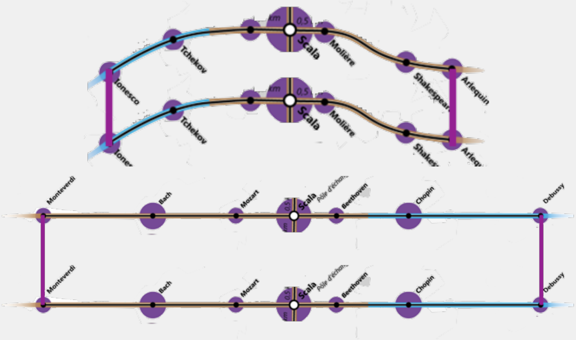
2. Each line is considered duplicating stations → PLATFORMS!!



Parameters : Network

- $[0, T]$: Time horizon.

- $L = LS \cup LSN$: Set of lines in the network formed by the set of lines containing short-turns and the set of lines that do not contain short-turns.
- $N_\ell = \{1, \ldots, n_\ell\}$: Stations of line $\ell \in L$.
- $S_\ell = \{1_{S_\ell}, \ldots, n_{S_\ell}\}$: Stations of short-turns $\ell \in LS$.
- d_i^ℓ: Travel distance between the stations i and $i+1$ of the line $\ell \in L$.
- e_i^ℓ: Stopping time that a train spends in the station i of the line $\ell \in L$.

- $Q = \{q_1, \ldots, q_{|Q|}\}$: Possible capacities for trains operating in all the lines.
- IS^ℓ: Safety interval between consecutive rounds in line $\ell \in L$.
- $K_\ell = \{1, \ldots, \bar{k}_\ell\}$: Rounds made in the line $\ell \in L$.

(Maximum number of rounds:$\bar{k}_\ell = \frac{T}{IS^\ell}$)

Parameters: Passengers Flow

- $\beta_{0_i}^{\ell}$: Passenger at the beginning of the time horizon at station i of line ℓ.
- β_i^{ℓ}: Rate of external passenger which enter to the transportation system at station i to use line ℓ.
- p_{ij}^{ℓ}: Proportion of passengers using the network starting at station i that go to the station j of line ℓ.
- $\tau_i^{\ell\ell'}$: Proportion of passengers that get off a train in a transfer-station i of the line ℓ to transfer to line ℓ'.

Parameters: Costs and profits

- b_q^ℓ: Fixed cost per complete line round of capacity $q \in Q$ on line $\ell \in L$. Largest capacities and largest lines usually involve more cost on the rounds.
- bS_q^ℓ: Fixed cost per short-turns round of capacity $q \in Q$ on line $\ell \in L$.
- γ_{ij}^ℓ: Unitary profit of transporting a passenger from the station i to the station j of the line $\ell \in L$.
- μ_1: Unitary penalty for passengers who cannot get on the first arriving train due to its limited capacity and still insist on using the system.
- μ_2: Unitary penalty for passengers who leave the system after they cannot get on the first arriving train due to its limited capacity.
- α: Proportion of passengers who decide to wait for the next train in case they cannot get on a train because of lack of capacity.

Variables

ℓ: line, k: round, i: station/platform, q: capacity.

Variable	Description
$t_1^{k\ell}$	Departure time from the initial station of line $\ell \in L$ at its k-th trip.
$f_i^{k\ell}$	Flow of passengers captured in the station i by the train that covers the k-th trip of the line $\ell \in L$, when k is a whole trip.
$g_i^{k\ell}$	Flow of passengers captured in the station $i \in S_\ell \backslash \{n_\ell\}$ by the train that covers the k-th trip of the line $\ell \in LS$, when k only covers the short-turn.
$w^{k\ell}$	Difference between the actual departure time from the first short-turn station of the k-th trip of line $\ell \in LS$ and the time when it should depart from this station regarding its departure time from the initial line station.
$y_q^{k\ell}$	$\begin{cases} 1 & \text{if the } k\text{-th trip of line } \ell \in L \text{ is a whole trip with capacity } q \\ 0 & \text{otherwise} \end{cases}$
$y_{Sq}^{k\ell}$	$\begin{cases} 1 & \text{if the } k\text{-th trip of line } \ell \in LS \text{ traverses the short-turn with capacity } q. \\ 0 & \text{otherwise} \end{cases}$
$t_i^{k\ell}$	Departure time from the station i of line ℓ in its k-th trip.
$D_i^\ell(t)$	Number of passengers accumulated from instant 0 up to instant t in the station i of line $\ell \in L$.
$h_i^{k\ell}$	Excess of passengers that where not able to get on the train at station i at the k-th trip of line $\ell \in L$ because of a lack of capacity.
$x_i^{k\ell}$	Excess of passengers only if k is a *true* trip for station i of line $\ell \in L$.

Variables: Auxiliary

- $t_i^{k\ell}$: Time instant in which a train departs from station i.

$$t_i^{k\ell} = t_1^{k\ell} + \sum_{r=1}^{i-1}(d_r^\ell + e_{r+1}^\ell), \quad i > 1,\ (i,\ell) \in \overline{S},\ k \in K_\ell, \qquad (\mathrm{T}-1)$$

$$t_i^{k\ell} = t_1^{k\ell} + \sum_{r=1}^{i-1}(d_r^\ell + e_{r+1}^\ell) + w^{k\ell}, \quad i > 1,\ i \in S_\ell,\ k \in K_\ell,\ \ell \in LS. \qquad (\mathrm{T}-2)$$

- $t_{1 \mapsto 1_{S_l}}$: Time difference between the time instant in which a train departs from the first station of the short-turns and the first station of the line.

$$t_{1 \mapsto 1_{S_l}} = \sum_{r=1}^{1_{S_l}-1}(d_r^l + e_{r+1}^\ell)$$

- $D_i^\ell(t)$: Accumulated flow of passengers up to time t at station i.

Variables: Auxiliary

- $h_i^{k\ell}$: Excess of passengers at station i.

- For the first round $(k = 1)$:

$$h_i^{1\ell} = D_i^{\ell}(t_i^{1\ell}) - f_i^{1\ell}, \quad \text{for } (i, \ell) \in \overline{S}, \qquad (\mathrm{H}-1)$$

$$h_i^{1\ell} = D_i^{\ell}(t_i^{1\ell}) - f_i^{1\ell} - g_i^{1\ell}, \quad \text{for } i \in S_\ell \backslash \{n_{S_\ell}\},\ \ell \in LS, \qquad (\mathrm{H}-2)$$

$$h_{n_{S_\ell}}^{1\ell} = D_{n_{S_\ell}}^{\ell}(t_{n_{S_\ell}}^{1\ell}) - f_{n_{S_\ell}}^{1\ell} + \sum_{r=1_{S_\ell}}^{n_{S_\ell}-1} \sum_{j=n_{S_\ell}+1}^{n_\ell} p_{rj}\, g_r^{1\ell}, \quad \text{for } \ell \in LS, \qquad (\mathrm{H}-3)$$

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- For round $k > 1$:

$$h_i^{k\ell} = D_i^\ell(t_i^{k\ell}) - D_i^\ell(t_i^{(k-1)\ell}) + \alpha h_i^{(k-1)\ell} - f_i^{k\ell}, \quad (i,\ell) \in \overline{\mathcal{S}} \tag{H - 4}$$

$$h_i^{k\ell} = D_i^\ell(t_i^{k\ell}) - D_i^\ell(t_i^{(k-1)\ell}) + \alpha h_i^{(k-1)\ell} - f_i^{k\ell} - g_i^{k\ell}, \quad i \in S_\ell \backslash\ \{n_{S_\ell}\}, \ell \in LS, \tag{H - 5}$$

$$h_{n_{S_\ell}}^{k\ell} = D_{n_{S_\ell}}^\ell(t_{n_{S_\ell}}^{k\ell}) - D_{n_{S_\ell}}^\ell(t_{n_{S_\ell}}^{(k-1)\ell}) + \alpha h_{n_{S_\ell}}^{(k-1)\ell} - f_{n_{S_\ell}}^{k\ell} + \sum_{r=1_{S_\ell}}^{n_{S_\ell}-1} \sum_{j=n_{S_\ell}+1}^{n_\ell} p_{rj}\, g_r^{k\ell},\ \ell \in LS. \tag{H - 6}$$

Variables: Semicontinuous

- Excess of passengers only for *true trips*

$$x_i^{k\ell} = \begin{cases} h_i^{k\ell} & \text{if } k \text{ is a true trip for station } i \text{ of line } \ell \\ 0 & \text{otherwise,} \end{cases} \qquad k \in K_\ell,\ i \in N_\ell,\ \ell \in L.$$

Objective Function: Minimize

Capacity Costs

$$\begin{cases} \sum_{k \in K_\ell} \sum_{q \in Q} b_q^\ell y_q^{k\ell} & \text{if } \ell \in LNS, \\ \sum_{k \in K_\ell} \sum_{q \in Q} b_q^\ell y_q^{k\ell} + \sum_{k \in K_\ell} \sum_{q \in Q} b_{Sq}^\ell (y_{Sq}^{k\ell} - y_q^{k\ell}) & \text{if } \ell \in LS. \end{cases} \tag{Cap(ℓ)}$$

$+$

Objective Function: Minimize

Reward per served passenger

$$
\begin{cases}
\displaystyle\sum_{i\in N_\ell\setminus\{1\}}\sum_{k\in K_\ell}\sum_{r=1}^{i-1}\gamma_{ri}^{\ell}p_{ri}^{\ell}f_r^{k\ell} & \text{if } \ell\in LNS,\\
\displaystyle\sum_{k\in K_\ell}\left(\sum_{i\in N_\ell\setminus\{1\}}\sum_{r=1}^{i-1}\gamma_{ri}^{\ell}p_{ri}^{\ell}f_r^{k\ell}+\sum_{i\in S_\ell\setminus\{1_{S_\ell}\}}\sum_{r=1_{S_\ell}}^{i-1}\gamma_{ri}^{\ell}p_{ri}^{\ell}g_r^{k\ell}+\sum_{\substack{r\in S_\ell:\\ r\neq n_{S_\ell}}}\sum_{j=n_{S_\ell}+1}^{n_\ell}\gamma_{rn_{S_\ell}}^{\ell}p_{rj}^{\ell}g_r^{k\ell}\right) & \text{if } \ell\in LS.
\end{cases}
\tag{RewPPass(ℓ)}
$$

$-$

Objective Function: Minimize

Cost NonServed Passengers

$$\alpha\mu_1 \sum_{i\in N_\ell} \sum_{k\in K_\ell} x_i^{k\ell} + (1-\alpha)\mu_2 \sum_{i\in N_\ell} \sum_{k\in K_\ell} x_i^{k\ell}, \qquad (\text{NonServed}(\ell))$$

Overall Cost:

$$(\text{Cap}(\ell)) - (\text{RewPPass}(\ell)) + (\text{NonServed}(\ell)) \qquad (\text{COST}(\ell))$$

Constraints: Capacities and true/fake trips

- For $\ell \in L$:

$$\sum_{q \in Q} y_q^{1\ell} = 1, \qquad \ell \in LNS, \qquad \text{(C1 - 1)}$$

$$\sum_{q \in Q} y_q^{k\ell} \leq 1, \qquad 1 < k < \bar{k}_\ell, \ell \in L, \qquad \text{(C1 - 2)}$$

$$\sum_{q \in Q} y_q^{\bar{k}_\ell \ell} = 1, \qquad \ell \in L, \qquad \text{(C1 - 3)}$$

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$$\sum_{q \in Q} y_q^{\bar{k}_\ell \ell} = 1, \qquad \ell \in L, \qquad \text{(C1 − 3)}$$

- For $\ell \in LS$:

$$y_q^{k\ell} \leq y_{Sq}^{k\ell}, \qquad q \in Q, k \in K_\ell, \ell \in LS, \qquad \text{(C1 − 4)}$$

$$\sum_{q \in Q} y_q^{1\ell} + \sum_{q \in Q} y_{Sq}^{1\ell} \geq 1, \qquad \ell \in LS, \qquad \text{(C1 − 5)}$$

$$\sum_{q \in Q} y_q^{\kappa_\ell \ell} = \sum_{q \in Q} y_{Sq}^{1\ell} - \sum_{q \in Q} y_q^{1\ell}, \qquad \ell \in LS, \qquad \text{(C1 − 6)}$$

$$\sum_{q \in Q} y_{Sq}^{k\ell} \leq 1, \qquad k \in K_\ell, \ell \in LS, \qquad \text{(C1 − 7)}$$

Constraints: Time Control

- For $\ell \in L$:

$$t_1^{1\ell} = 0, \qquad t_1^{\bar{k}\ell} = T, \qquad \ell \in L, \tag{C2 − 1}$$

$$IS\left(\sum_{q \in Q} y_q^{k\ell}\right) \leq t_i^{k\ell} - t_i^{(k-1)\ell} \leq T\left(\sum_{q \in Q} y_q^{k\ell}\right), \quad k > 1, (i, \ell) \in \overline{S} \tag{C2 − 2}$$

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- For $\ell \in LS$:

$$IS\left(\sum_{q\in Q} y_{Sq}^{k\ell}\right) \le t_i^{k\ell} - t_i^{(k-1)\ell} \le (T + t_{1\mapsto 1_{S_l}})\left(\sum_{q\in Q} y_{Sq}^{k\ell}\right), i \in S_\ell, k > 1, \tag{C2 − 3}$$

$$t_1^{\kappa_\ell \ell} \le T\left(1 - \sum_{q\in Q} y_{Sq}^{1\ell} + \sum_{q\in Q} y_q^{1\ell}\right), \tag{C2 − 4}$$

$$-t_{1\mapsto 1_{S_l}}\left(1 - \sum_{q\in Q_\ell} y_q^{k\ell}\right) \le w^{k\ell} \le (T + t_{1\mapsto 1_{S_l}})\left(1 - \sum_{q\in Q_\ell} y_q^{k\ell}\right), k \in K_\ell, \tag{C2 − 5}$$

Constraints: Flow Control

- Flow determined by the capacity of the train:

$$f_i^{k\ell} + \sum_{r=1}^{i-1} f_r^{k\ell} \left(\sum_{j=i+1}^{n_\ell} p_{rj}^{\ell} \right) \leq \sum_{q \in Q} q y_q^{k\ell}, \quad k \in K_\ell, i \in N_\ell, \ell \in L, \qquad (C3-1)$$

$$g_i^{k\ell} + \sum_{r=1_{S_\ell}}^{i-1} g_r^{k\ell} \left(\sum_{j=i+1}^{n_{S_\ell}} p_{rj}^{\ell} \right) \leq \sum_{q \in Q} q(y_{Sq}^{k\ell} - y_q^{k\ell}), \; k \in K_\ell, i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS, \qquad (C3-2)$$

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$$g_i^{k\ell} + \sum_{r=1_{S_\ell}}^{i-1} g_r^{k\ell} \left(\sum_{j=i+1}^{n_{S_\ell}} p_{rj}^{\ell} \right) \leq \sum_{q \in Q} q (y_{Sq}^{k\ell} - y_q^{k\ell}), \; k \in K_\ell, i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS, \qquad (\text{C3}-2)$$

- Flow determined by the demand function:

$$f_i^{1\ell} \leq D_i^{\ell}(t_i^{1\ell}), \quad i \in N_\ell, \ell \in L, \qquad (\text{C3}-3)$$

$$f_i^{kl} \leq D_i^{\ell}(t_i^{k\ell}) - D_i^{\ell}(t_i^{(k-1)\ell}) + \alpha h_i^{(k-1)\ell}, \quad k > 1 i \in N_\ell, \ell \in L, \qquad (\text{C3}-4)$$

$$g_i^{1\ell} \leq D_i^{\ell}(t_i^{1\ell}), \;\; i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS, \qquad (\text{C3}-5)$$

$$g_i^{k\ell} \leq \left(D_i^{\ell}(t_i^{k\ell}) - D_i^{\ell}(t_i^{(k-1)\ell}) \right) + \alpha h_i^{(k-1)\ell}, \; k > 1, i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS. \qquad (\text{C3}-6)$$

Constraints: Passenger Surplus

$$x_i^{k\ell} \geq h_i^{k\ell} - M_i^{\ell}\left(1 - \sum_{q \in Q_\ell} y_q^{k\ell}\right), (i,l) \in \overline{S} \text{ or } (i = n_{S_\ell},\ \ell \in LS), \quad \text{(C4} - 1)$$

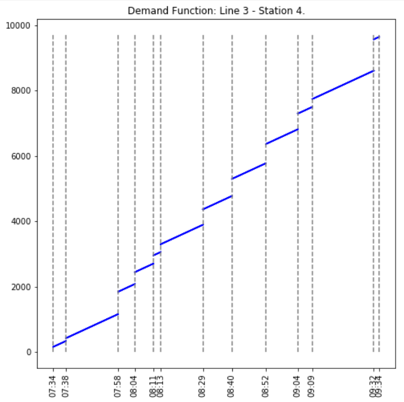
$$x_i^{k\ell} \geq h_i^{k\ell} - M_i^{\ell}\left(1 - \sum_{q \in Q_\ell} y_{Sq}^{k\ell}\right), i \in S_\ell \backslash \{n_{S_\ell}\}, \ell \in LS, \quad \text{(C4} - 2)$$

A Math Programming Formulation

$$
\begin{aligned}
&\min \sum_{\ell \in L} \mathrm{COST}(\ell) \\
&\text{s.t. } (\mathrm{C1}),\ (\mathrm{C2}),\ (\mathrm{C3}) \text{ and } (\mathrm{C4}), \\
&\quad 0 \le t_1^{k\ell} \le T, && k \in K_\ell,\ \ell \in L, \\
&\quad f_i^{k\ell} \ge 0, && k \in K_\ell,\ i \in N_\ell,\ \ell \in L, \qquad (\mathrm{P}) \\
&\quad g_i^{k\ell} \ge 0, && k \in K_\ell,\ i \in S_\ell\, \ell \in LS, \\
&\quad w^{k\ell} \in \mathbb{R}, && k \in K_\ell,\ \ell \in LS, \\
&\quad x_i^{k\ell} \ge 0, && k \in K_\ell,\ i \in N_\ell,\ \ell \in L, \\
&\quad y_q^{k\ell} \in \{0,1\}, && k \in K_\ell,\ q \in Q,\ \ell \in L, \\
&\quad y_{S_q}^{k\ell} \in \{0,1\}, && k \in K_\ell,\ q \in Q, \ell \in LS.
\end{aligned}
$$

The Demand function

$$D_i^\ell(t) = \beta_{0i}^\ell + \beta_i^\ell t + J_{i\ell}^E(t) + \sum_{\ell' \neq \ell, \ell' \ni i} J_{i\ell\ell'}^I(t), \qquad \text{(D)}$$



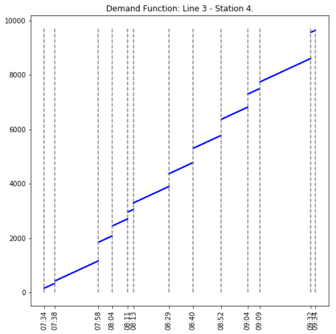
β_{0i}^ℓ: Number of passengers awaiting in the station i at the beginning of the planning horizon.

β_i^ℓ: Average rate of passengers arriving to the station i by unit of time.

$J_{i\ell}^E(t)$: Sum of the external block of arrivals of passengers up to the instant t to the station i.

$J_{i\ell\ell'}^I(t)$: Sum of the block arrivals of passengers up to the instant t to the interchange station i of line $\ell \in L$ from line $\ell' \in L$.

The Demand function



$se_r^{i\ell}$: Time instants when the block of arrivals occur ($r = 0, \ldots, re^{i\ell}$).

$\Psi^{\ell}_{ir'}$: Discontinuity flow jump produced at time instant $se_r^{i\ell}$.

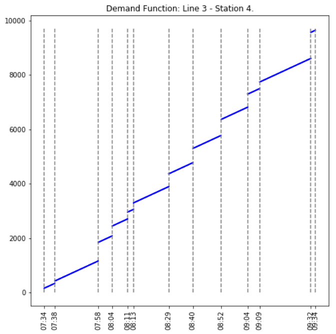
$$\delta^E_{ri\ell}(t) = \begin{cases} 1 & \text{if } t \in [se_r^{i\ell}, se_{r+1}^{i\ell}), \\ 0 & \text{otherwise}, \end{cases}$$

$$se_r^{i\ell}\delta^E_{ri\ell}(t) \le t < se_{r+1}^{i\ell}\delta^E_{ri\ell}(t) + \widehat{T}_\ell(1 - \delta^E_{ri\ell}(t)),$$

$$\sum_{r=0}^{re^{i\ell}} \delta^E_{ri\ell}(t) = 1,$$

- External Arrivals: $J^E_{i\ell}(t) = \sum_{r=0}^{re^{i\ell}} \left(\sum_{r' \le r} \Psi^{\ell}_{ir'} \right) \delta^E_{ri\ell}(t), \quad i \in N_\ell,\ \ell \in L.$

The Demand function



$se_r^{i\ell}$: Time instants when the block of arrivals occur ($r = 0, \ldots, re^{i\ell}$).

$\Psi_{ir'}^{\ell}$: Discontinuity flow jump produced at time instant $se_r^{i\ell}$.

$$\delta_{ri\ell}^{E}(t) = \begin{cases} 1 & \text{if } t \in [se_r^{i\ell}, se_{r+1}^{i\ell}), \\ 0 & \text{otherwise}, \end{cases}$$

$$se_r^{i\ell}\delta_{ri\ell}^{E}(t) \leq t < se_{r+1}^{i\ell}\delta_{ri\ell}^{E}(t) + \widehat{T}_\ell(1 - \delta_{ri\ell}^{E}(t)),$$

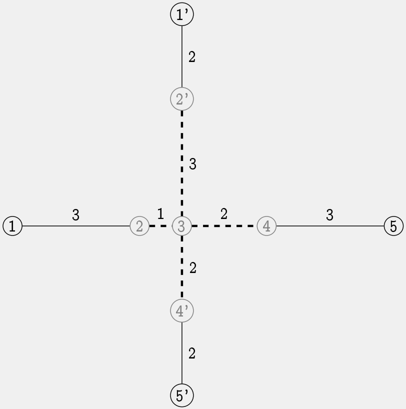
$$\sum_{r=0}^{re^{i\ell}} \delta_{ri\ell}^{E}(t) = 1,$$

- External Arrivals: $J_{i\ell}^{E}(t) = \sum_{r=0}^{re^{i\ell}} \left(\sum_{r' \leq r} \Psi_{ir'}^{\ell} \right) \delta_{ri\ell}^{E}(t), \quad i \in N_\ell,\ \ell \in L.$
- Internal Arrivals:

$$J_{i\ell\ell'}^{I}(t) = \sum_{r=0}^{\bar{k}_{\ell'}} \left(\sum_{r' \leq r} \Phi_{ir'}^{\ell\ell'} \right) \delta_{ri\ell\ell'}^{I}(t), \quad i \in N_\ell \cap N_{\ell'},\ \ell \in L, \ell' \in L.$$

Example (A simplified version of Metrolab network)

Dimensions: Capacities: 800 and 1600, $T = 20$ min, $\bar{k}_\ell = 7$ and 10.



Example

IS^{ℓ}: 2 minutes, $\alpha = 1$, $\mu_1 = 0.1875$, $\tau = 0.4$.

p_{ij}^{ℓ}	1	2	3	4	5
1	0	0.40	0.35	0.20	0
2	0.40	0	0.60	0.35	0
3	0.35	0.6	0	0.95	0
4	0.20	0.35	0.95	0	1
5	0.05	0.05	0.05	1	0

p_{ij}^{ℓ}	1'	2'	3	4'	5'
1'	0	0.40	0.35	0.20	0
2'	0.40	0	0.60	0.35	0
3	0.35	0.60	0	0.95	0
4'	0.20	0.35	0.95	0	1
5'	0.05	0.05	0.05	1	0

Table: O-D matrix of Example: Lines 1-2 (left) and 3-4 (right).

γ_{ij}^{ℓ}	1	2	3	4	5
1	0	0.3	0.4	0.6	1
2	0.3	0	0.1	0.3	1
3	0.5	0.2	0	0.2	1
4	0.6	0.3	0.1	0	0
5	0.9	0.6	0.4	0.3	0

γ_{ij}^{ℓ}	1'	2'	3	4'	5'
1'	0	0.2	0.5	0.7	1
2'	0.2	0	0.3	0.5	1
3	0.4	0.2	0	0.2	0
4'	0.7	0.5	0.3	0	0
5'	0.9	0.7	0.5	0.2	0

Table: Rewards of Example: Lines 1-2 (left) and 3-4 (right).

Example

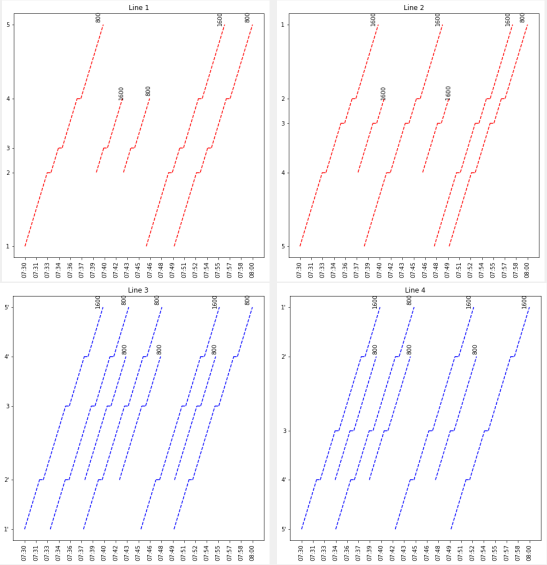
	Lines									
	$\ell = 1$					$\ell = 2$				
Stations (i)	1	2	3	4	5	5	4	3	2	1
β_{0i}^{ℓ}	50	50	50	50	0	50	50	50	50	0
β_{i}^{ℓ}	10	100	120	90	0	10	160	180	150	0
	$\ell = 3$					$\ell = 4$				
Stations (i)	1'	2'	3	4'	5'	5'	4'	3	2'	1'
β_{0i}^{ℓ}	50	50	50	50	50	50	50	50	50	50
β_{i}^{ℓ}	10	150	170	160	0	10	100	180	150	0

Table: Coefficients of the Demand functions of Example.

Model Coded in `Python 3.6` + `Gurobi 8.0` in a Mac OSX with an Intel Core i7 processor at 3300 MHz and 16GB of RAM.

CPU: 12 hours MIP GAP: 1.51%.

Example

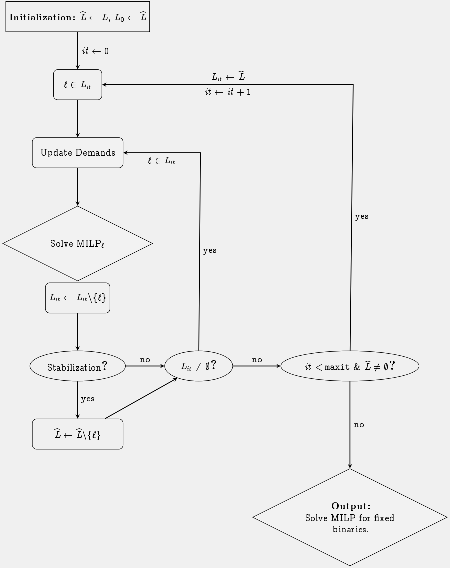

Line 1
Line 2
Line 3
Line 4
800
1600
1
2
3
4
5
1'
2'
3'
4'
5'
07:30
07:31
07:33
07:34
07:36
07:37
07:39
07:40
07:42
07:43
07:45
07:46
07:48
07:49
07:51
07:52
07:54
07:55
07:57
07:58
08:00

Timetable (Line 1)

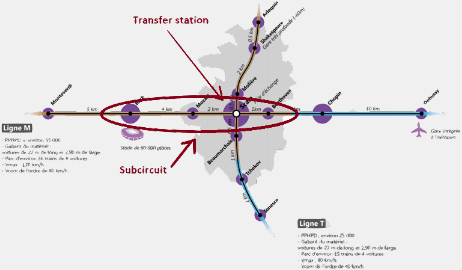
k: Capacity	i	DepTime	Get-Off	$f_i^{k\ell}$ ($g_i^{k\ell}$)	$h_i^{k\ell}$	$x_i^{k\ell}$	Load
1: 800	1	07:30:00	0.00	50.00	0.00	0.00	50.00
	2	07:33:30	20.00	400.00	0.00	0.00	430.00
	3	07:35:00	257.50	627.50	101.50	101.50	800.00
	4	07:37:30	746.13	725.00	0.00	0.00	778.88
	5	07:40:30	778.88	0.00	0.00	0.00	0.00
2S: 1600	2	07:39:34	0.00	606.94	0.00	0.00	606.94
	3	07:41:04	364.17	1231.59	0.00	0.00	1474.36
	4	07:43:34	1474.36	0.00	638.18	0.00	0.00
3: **0**	1	07:30:00	0.00	0.00	0.00	0.00	0.00
	2	07:39:34	0.00	0.00	0.00	0.00	0.00
	3	07:41:04	0.00	0.00	0.00	0.00	0.00
	4	07:43:34	0.00	0.00	638.18	0.00	0.00
	5	07:40:30	0.00	0.00	0.00	0.00	0.00
4S: 800	2	07:43:12	0.00	364.02	0.00	0.00	364.02
	3	07:44:42	218.41	603.05	0.00	0.00	748.66
	4	07:47:12	748.66	0.00	1014.15	0.00	0.00
5: **0**	1	07:30:00	0.00	0.00	0.00	0.00	0.00
	2	07:43:12	0.00	0.00	0.00	0.00	0.00
	3	07:44:42	0.00	0.00	0.00	0.00	0.00
	4	07:47:12	0.00	0.00	1014.15	0.00	0.00
	5	07:40:30	0.00	0.00	0.00	0.00	0.00
6: 1600	1	07:46:15	0.00	162.60	0.00	0.00	162.60
	2	07:49:45	65.04	655.05	0.00	0.00	752.61
	3	07:51:15	449.94	1297.33	0.00	0.00	1600.00
	4	07:53:45	1494.25	1494.25	109.44	109.44	1600.00
	5	07:56:45	1600.00	0.00	0.00	0.00	0.00
7: 800	1	07:50:00	0.00	37.40	0.00	0.00	37.40
	2	07:53:30	14.96	373.99	0.00	0.00	396.43
	3	07:55:00	237.48	641.05	0.00	0.00	800.00
	4	07:57:30	747.38	446.03	0.00	0.00	498.65
	5	08:00:30	498.65	0.00	0.00	0.00	0.00

Math-Heuristic Algorithm

Divide and conquer



Case Study



q	b_q^ℓ & bS_q^ℓ
400	48.8$\times$ length(ℓ)
800	97.4$\times$ length(ℓ)
1600	194.8$\times$ length(ℓ)

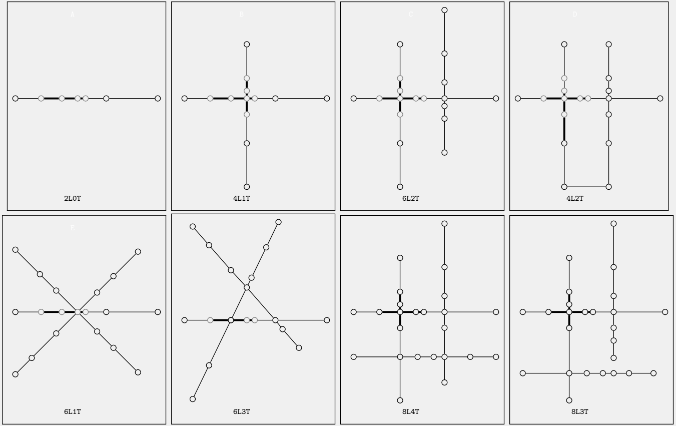
Line	μ	IS^ℓ
M	0.1875	1.66
T	0.1875	1.25

$\star\ \gamma_{ij}^\ell = 0.075 \times d_{ij}^\ell + 0.075$

Time window: 7:30 - 9:30 a.m.

Line M					Line T				
	d_i^ℓ	e_i^ℓ	$\overleftarrow{\beta}_i^\ell$	$\overrightarrow{\beta}_i^\ell$		d_i^ℓ	e_i^ℓ	$\overleftarrow{\beta}_i^\ell$	$\overrightarrow{\beta}_i^\ell$
Debussy - Chopin	6.32	0	3.3	138.7	Arlequin-Shak.	0.88	0	3.3	100
Chopin-Beeth.	2.53	0.5	36.7	130	Shak.-Molière	2.22	0.3	5	93.3
Beeth.-Scala	0.97	0.3	108.3	21.7	Molière-Scala	0.88	0.3	38.3	68.3
Scala-Mozart	1.94	0.6	125	99.6	Scala-Beaum.	0.88	0.5	25	140
Mozart Bach	2.53	0.3	120	13	Beaum.-Tchekov	1.11	0.3	50	41.6
Bach-Mont.	3.16	0.5	40	8.7	Tchekov-Ionescu	1.11	0.3	53.3	3.33
Max Rounds			40	30	Max Rounds			20	40

More Experiments



More Experiments

Network	$\lvert LS \rvert$	Matheuristic		MILP		
		BestObj	**CPU**(sec.)	**BestObj**	**CPU** (sec.)	**GAP** (%)
2L0T	0	145702	< 0.1	145573	7	0.09
	2	114145	11	112916	TL	1.08
4L1T	0	206729	132	206242	TL	0.24
	4	152890	631	152016	TL	0.57
4L2T	0	348267	3522	347224	TL	0.30
	4	333102	4892	332665	TL	0.13
6L1T	0	276961	1130	276545	TL	0.15
	2	235488	1521	234589	TL	0.38
6L2T	0	249038	606	248854	TL	0.07
	4	203080	2882	203080	TL	0.00
6L3T	0	248988	520	248979	TL	< 0.01
	2	217004	2688	216909	TL	0.04
8L3T	0	404627	432	404147	TL	0.12
	4	362916	1467	362908	TL	< 0.01
8L4T	0	404469	1191	404211	TL	0.06
	4	374067	1144	374064	TL	< 0.01

TL= 12 hours.

Extensions

1. Strategies to put join time windows.
2. Introduce the average speed between consecutive stations and the stopping time that a train spends in a station as variables.
3. Flexibilize the use of short-turns.
4. Consider stochastic demands.
5. ...

Extensions: Public Transportation Planning

1. **network design**, where the stations, links and routes of the lines are established,
2. **line planning**, specifying the frequency and the capacity of the vehicles used in each line.
3. **timetabling**, defining the arrival/departure times and
4. **scheduling**, in which vehicles and/or crews are planned.

A very complex problem: multiobjective, multilevel, stochastic, combinatorial, ...

Muchas gracias.