Mathematical models and exact algorithms for the Hospitals / Residents problem with Couples and Ties

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V Jornadas Doctorales del Programa de Doctorado en Matemáticas de la UCA, 2019

Outline

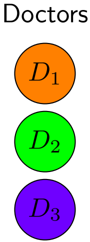
Scrubs

Problem Definition

The Hospital-Resident Problem (HR)

Given

- a set of n_d **junior doctors**,



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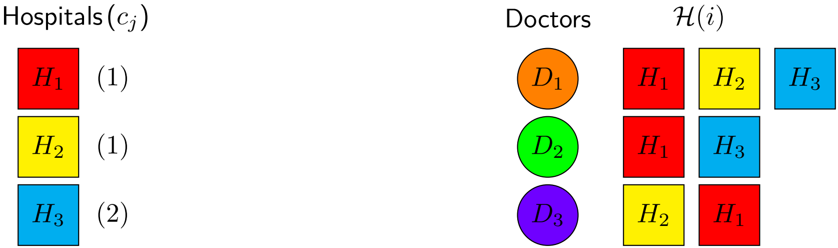

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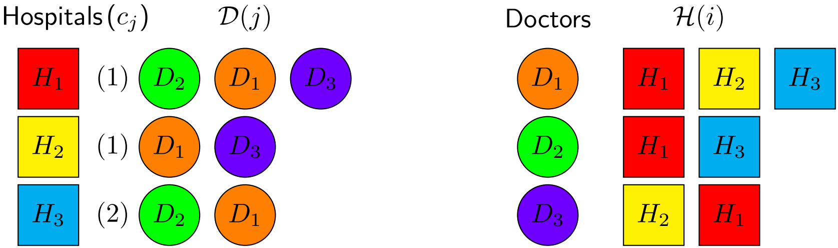

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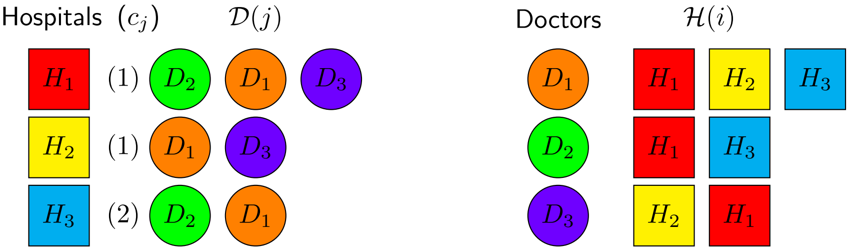

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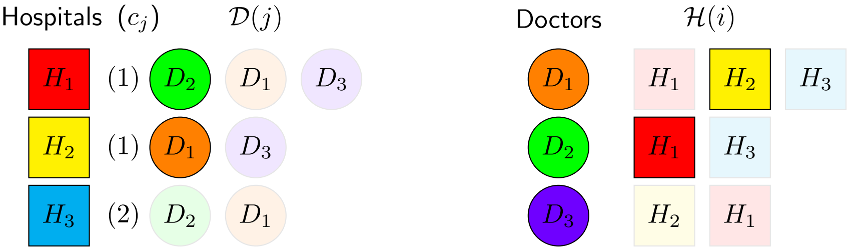

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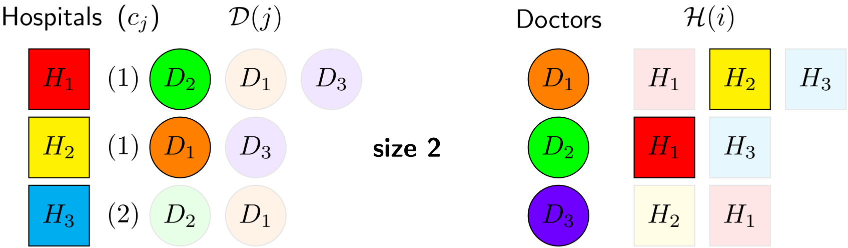

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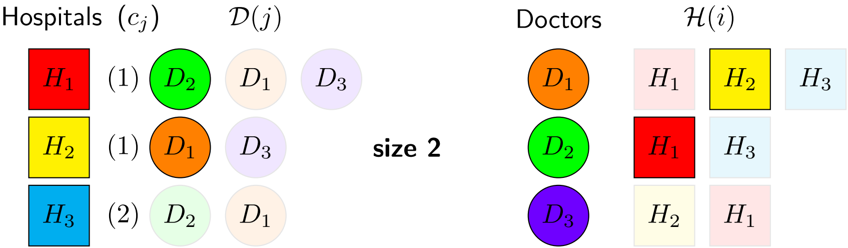

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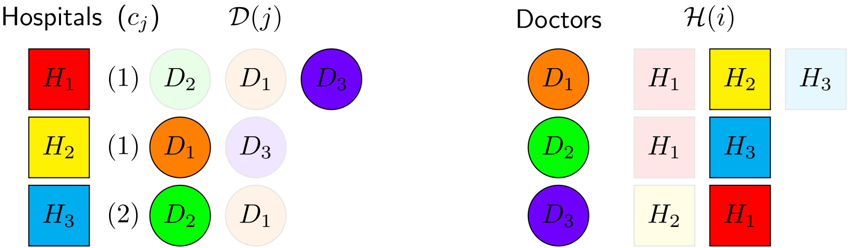

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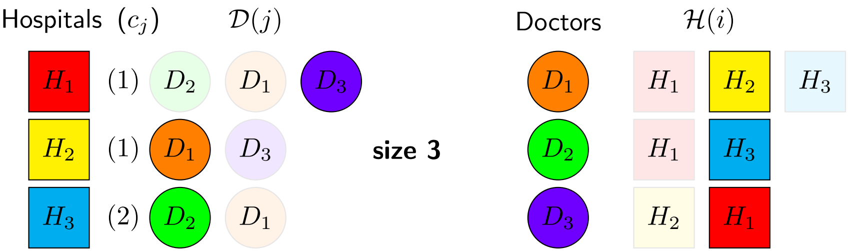

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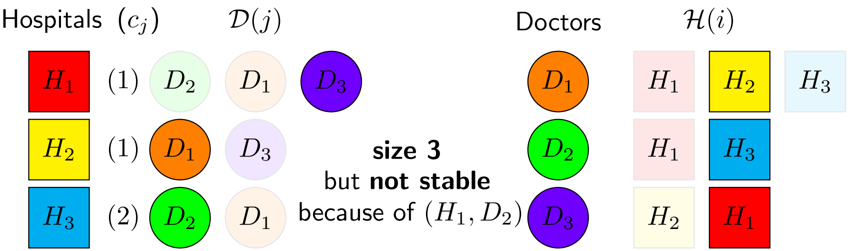

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Find **matching**, i.e., an **assignment** of doctors to hospitals, of

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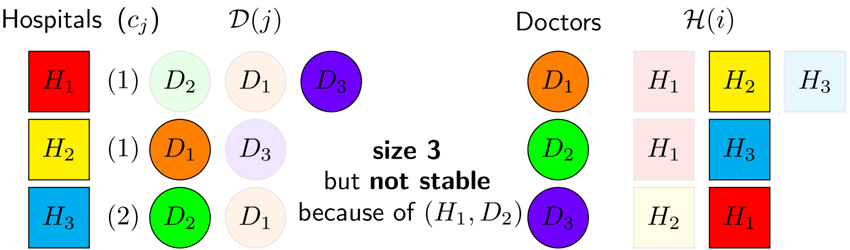
- ensures **stability**, i.e., no doctor-hospital pair prefer to be matched together than remain with their current assignee/s (if any).



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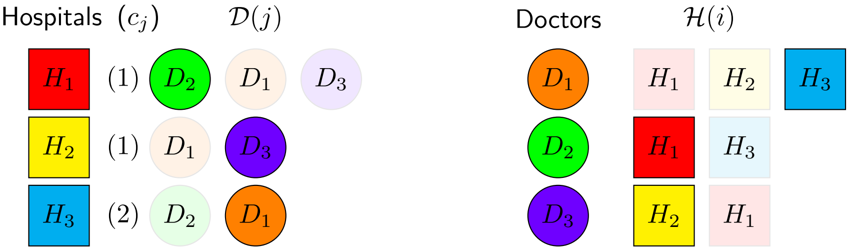

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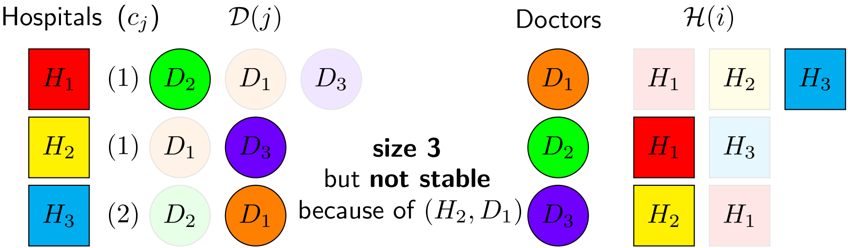

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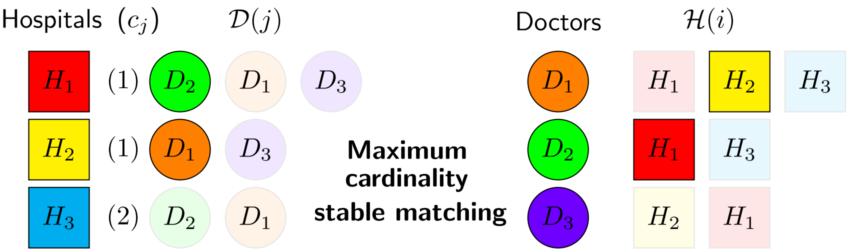

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Gale and Shapley (1962)

- A stable matching **always exists**.
- All stable matchings have the **same size**.
- A stable matching (of maximum cardinality) can be found in **polynomial time**.

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Ties occur if

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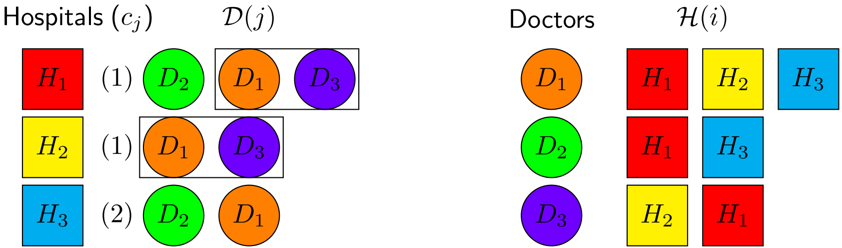

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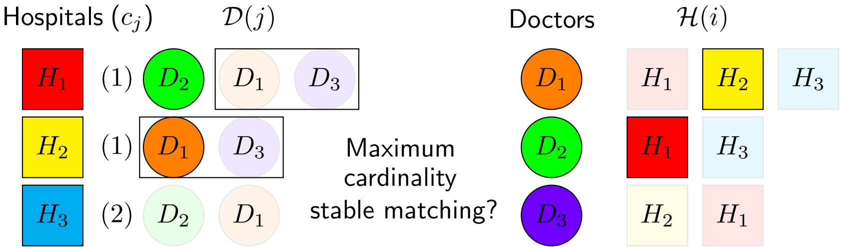

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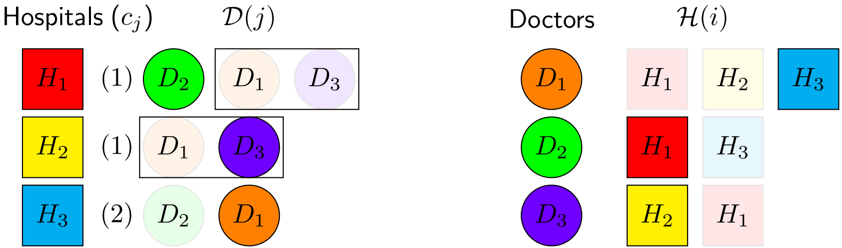

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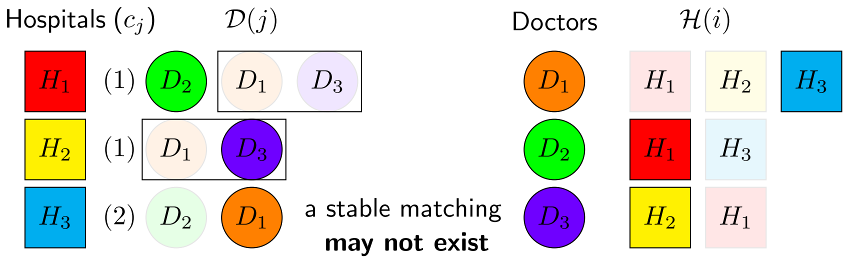

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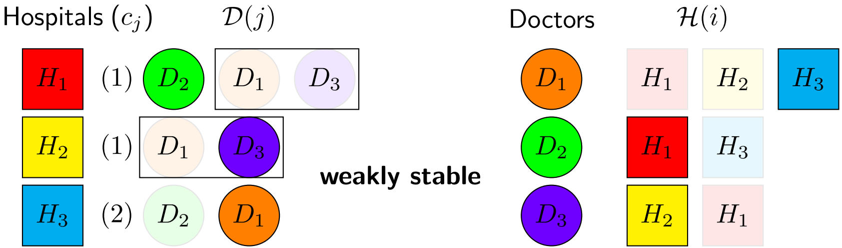

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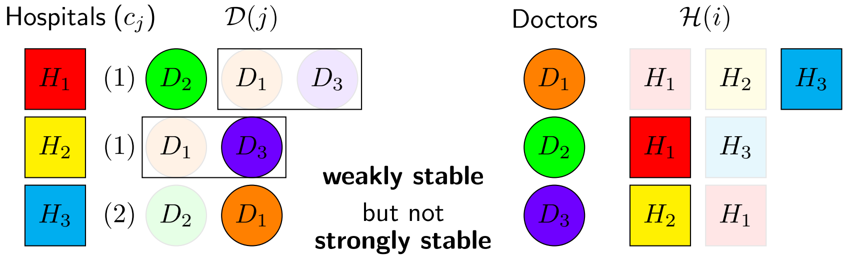

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A pair (i, j) is called a **blocking pair** if

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- Stable matchings can have **different sizes**.
- Finding a **maximum cardinality** stable matching is **NP-hard**.

Mathematical Formulation for HRT

Given

- n_h hospitals and n_d junior doctors,
- a ranked list of acceptable doctors for each hospital $\mathcal{D}(j)$,
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Decision variables

$$x_{ij} = \begin{cases} 1 & \text{if doctor } i \text{ is \textbf{assigned} to hospital } j \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, \ldots, n_d$, and $j \in \mathcal{H}(i)$.

Mathematical Formulation for HRT

Integer Linear Programming Formulation (Kwanashie & Manlove (2013))

$$\max \quad \sum_{i=1}^{n_d} \sum_{j \in \mathcal{H}(i)} x_{ij}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{H}(i)} x_{ij} \leq 1 \qquad i = 1, \ldots, n_d$$

$$\sum_{i \in \mathcal{D}(j)} x_{ij} \leq c_j \qquad j = 1, \ldots, n_h$$

$$c_j \left(1 - \sum_{q \in \mathcal{H}_j^{\leq}(i)} x_{iq} \right) \leq \sum_{p \in \mathcal{D}_i^{\leq}(j)} x_{pj} \qquad i = 1, \ldots, n_d, j \in \mathcal{H}(i)$$

$$x_{ij} \in \{0, 1\} \qquad i = 1, \ldots, n_d, j \in \mathcal{H}(i)$$

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Stability constraint

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New Mathematical Formulations for HRT

Idea

Replace the **sum of variables** x_{iq} and x_{pj} by the **dummy variables**

$$y^d_{ik} = \begin{cases} 1 & \text{if doctor } i \text{ is } \textbf{assigned} \text{ to hospital of } \textbf{rank at most } k \\ 0 & \text{otherwise} \end{cases}$$

$$y^h_{jk} = \text{the } \textbf{number} \text{ of doctors assigned to hosp. } j \text{ of } \textbf{rank at most } k$$

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Stability constraint

$$c_j(1 - y^d_{ik}) \leq y^h_{j,r^h_i(j)} \qquad i,\, k,\, j \in \mathcal{H}^{=}_k(i)$$

where

- $r^h_i(j)$ is the **rank** of doctor i in $\mathcal{D}(j)$, and
- $\mathcal{H}^{=}_k(i)$ the set of hospitals **at rank** k in $\mathcal{H}(i)$.

New Mathematical Formulations for HRT

Define $g^d(i)$ ($g^h(j)$) as the number of **distinct ranks** in $\mathcal{H}(i)$ ($\mathcal{D}(j)$).

New IP Formulation (Delorme et al. (2019))

$$\begin{aligned}
\max \quad & \sum_{i=1}^{n_d} \sum_{j \in \mathcal{H}(i)} x_{ij} & \\
\text{s.t.} \quad & \sum_{j \in \mathcal{H}_1^{=}(i)} x_{ij} = y_{i1}^d, & i = 1, \ldots, n_d \\
& \sum_{j \in \mathcal{H}_k^{=}(i)} x_{ij} + y_{i,k-1}^d = y_{ik}^d, & i = 1, \ldots, n_d, k = 2, \ldots, g^d(i) \\
& \sum_{i \in \mathcal{D}_1^{=}(j)} x_{ij} = y_{j1}^h, & j = 1, \ldots, n_h \\
& \sum_{i \in \mathcal{D}_k^{=}(j)} x_{ij} + y_{j,k-1}^h = y_{jk}^h, & j = 1, \ldots, n_h, k = 2, \ldots, g^h(j)
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New IP Formulation (Delorme et al. (2019))

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& x_{ij} \in \{0, 1\} && i = 1, \dots, n_d, j \in \mathcal{H}(i) \\
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For the ILP model, using dummy variables results in

- **no change** in the continuous relaxation value,
- an **increase** in the number of variables,
- an **increase** in the number of constraints,
- a **decrease** in the number of non-zero elements.

New Mathematical Formulations for HRT

Alternative Stability Constraints

Replace the stability constraints by an **alternative version** based on **ranks**.

Introduce **new binary variables**

$$z_{jk} = \begin{cases} 1 & \text{hospital } j \text{ is } \textbf{filled} \text{ by doctors of } \textbf{rank at most } k-1 \\ 0 & \text{otherwise} \end{cases}$$

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Stability constraint

$$1 \leq z_{jk} + y^d_{i,r^d_j(i)} \qquad j,\, k \geq 2,\, i \in \mathcal{D}^{=}_{k-1}(j)$$

where

- $r^d_j(i)$ is the **rank** of hospital j in $\mathcal{H}(i)$, and
- $\mathcal{D}^{=}_{k-1}(i)$ the set of hospitals **at rank** k in $\mathcal{H}(i)$.

New Mathematical Formulations for HRT

Using this alternative set of stability constraints results in

- **a modification** (usually an improvement) in the continuous relaxation,
- an **increase** in the number of variables,
- an **increase** in the number of constraints,
- a **decrease** in the number of non-zero elements.

New Mathematical Formulations for HRT

Further stability constraints modifications

- The **two versions** of stability constraints **do not dominate** each other, both can be used at the same time.

  Using **both sets** of stability constraints results in
  - an improvement in the continuous relaxation,
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New Mathematical Formulations for HRT

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  Using **both sets** of stability constraints results in
  - an improvement in the continuous relaxation,
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  - an increase in the number of constraints,
  - an increase in the number of non-zero elements.
- The two sets stability constraints **can be merged**.
  Merging stability constraints results in
  - a deterioration in the continuous relaxation,
  - no change in the number of variables,
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Instances solved in less than 3600 seconds using Gurobi 7.5.2

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	V2		3	10	744.5	2300	5014	16 495
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	V1, V2		3	16	744.3	2300	6912	75 971
Yes	V1, V2		3	5	744.3	4548	8345	19 676
	V1, V2	Yes	3	26	746.2	2300	5363	74 422
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	V2		3	10	744.5	2300	5014	16 495
Yes	V2		3	10	744.5	4548	6446	15 879
	V1, V2		3	16	744.3	2300	6912	75 971
Yes	V1, V2		3	5	744.3	4548	8345	19 676
	V1, V2	Yes	3	26	746.2	2300	5363	74 422
Yes	V1, V2	Yes	3	9	746.2	4548	6796	18 127

Computational experiments (1)

We used 3 **real world instances** with 750 doctors and 50 hospitals (Scottish junior doctor assignment for 2006-2008).

Instances solved in less than 3600 seconds using Gurobi 7.5.2

Dummy	Stab.	Merged	Opt	Time	Rel.	Var.	Cons.	N.Z.
	V1		3	145	747.4	1898	2714	63 272
Yes	V1		3	19	747.4	4146	4146	11 278
	V2		3	10	744.5	2300	5014	16 495
Yes	V2		3	10	744.5	4548	6446	15 879
	V1, V2		3	16	744.3	2300	6912	75 971
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	V1, V2	Yes	3	26	746.2	2300	5363	74 422
Yes	V1, V2	Yes	3	9	746.2	4548	6796	18 127

Computational experiments (2)

We used five sets of **randomly generated instances** with $\kappa \times 750$ doctors and $\kappa \times 50$ hospitals.

Computational experiments (2)

We used five sets of **randomly generated instances** with $\kappa \times 750$ doctors and $\kappa \times 50$ hospitals.

Number of randomly generated instances solved in less than 1800 seconds using Gurobi 6.5:

Method			$\kappa = 1$		$\kappa = 2$		$\kappa = 3$		$\kappa = 5$	
Dummy	Stab.	Merged	Opt	Time	Opt	Time	Opt	Time	Opt	Time
	V1		25	848	12	2656	2	3485	0	3600
Yes	V1		29	336	17	2056	9	2919	2	3524
	V2		30	82	16	1844	9	2667	2	3491
Yes	V2		30	136	18	1815	10	2694	2	3395
	V1, V2		30	66	25	1170	14	2298	1	3495
Yes	V1, V2		30	62	25	1070	13	2267	4	3372
	V1, V2	Yes	30	134	20	1702	13	2763	1	3509
Yes	V1, V2	Yes	30	52	25	1023	14	2184	2	3397

A Special Case: Child Adoption

Child Adoption

Assigning **children** to **foster families**.

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The **same algorithms** can be applied (hospitals = families, doctors = children) but

- family capacity is 1
- preference lists are longer (based on scores between each pair child/family)
- objective function is to find a maximum weighted matching

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In terms of **adaptation**

- dummy variables and constraint merging remains the same
- if merging is applied, an additional set of stability constraints can be derived

Computational experiments: Child Adoption

We used a set of 22 **real world instances** with 550 children and 894 families (provided by Coram charity).

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Number of instances solved in less than 3600 seconds using Gurobi 7.5.2

Dummy	Stab.	Merged	Opt	Time	Rel.	Var.	Cons.	N.Z.
	V1		21	434	42 966	94 764	96 208	39 152 977
Yes	V1		22	87	42 966	101 220	101 220	390 790
	V1	Yes	20	416	43 030	94 764	3457	13 619 285
Yes	V1	Yes	22	74	43 030	101 220	8468	298 038
	V2	Yes	20	722	43 010	94 764	7899	39 853 723
Yes	V2	Yes	22	66	43 010	101 220	12 911	397 245

Computational experiments: Child Adoption

Number of **randomly generated instances** with $\kappa \times 550$ children and $\kappa \times 894$ families solved in less than 3600 seconds using Gurobi 7.5.2

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Method			$\kappa = 1$					$\kappa = 2$				
Dum.	Sta.	Mer.	Opt	Time	Var.	Cons.	N.Z.	Opt	Time	Var.	Cons.	N.Z.
	V1		220	341	64	65	21.2	74	2702	359	361	278.3
Yes	V1		220	115	73	73	0.3	121	2185	380	380	1.5
	V1	Yes	220	202	64	4	6.7	52	2963	359	9	94.9
Yes	V1	Yes	220	84	73	11	0.2	134	2114	380	27	1.1
	V2	Yes	220	508	64	10	21.9	43	3047	359	24	282.4
Yes	V2	Yes	220	56	73	17	0.3	143	2107	380	42	1.5

Number of variables and constraints expressed in thousands
Number of non-zero elements expressed in millions

Outline

Problem definition

Hospital-Resident problem with Couples and Ties (HRCT)

Pairs of doctors can specify their **preferences as pair**.

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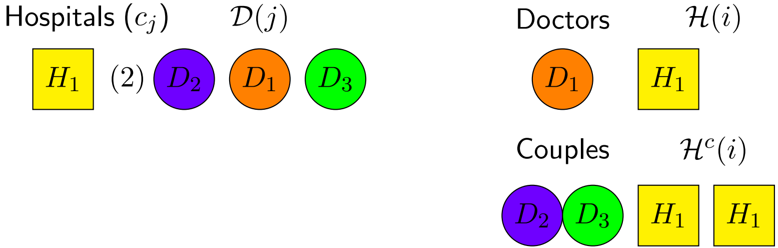

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Hospital-Resident problem with Couples and Ties (HRCT)

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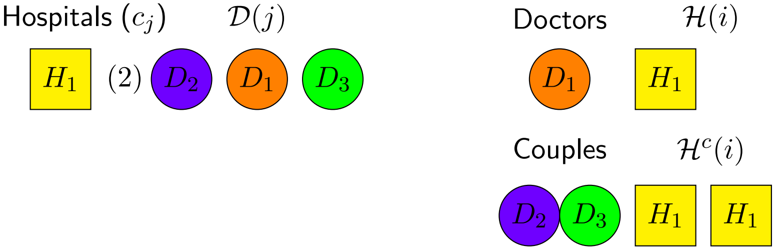
Given in addition

- a set of n_c **couples of junior doctors**,
- a ranked list $\mathcal{H}^c(i)$ of **acceptable pairs** of hospitals for each couple.



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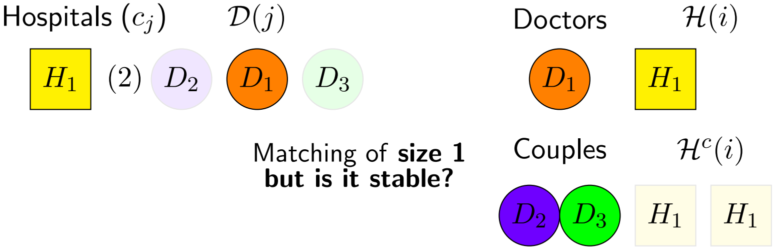

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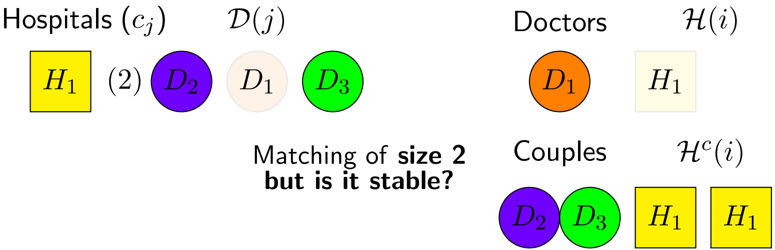

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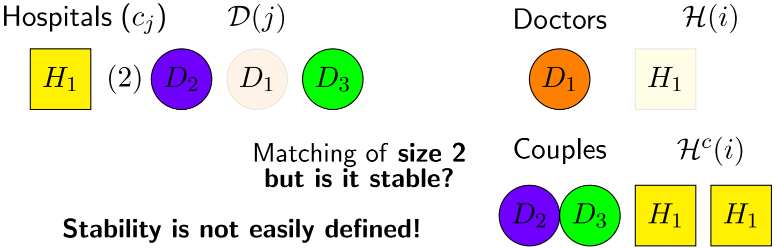

Matching of **size 2**
but is it stable?

Stability is not easily defined!

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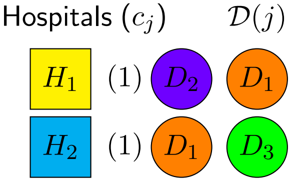

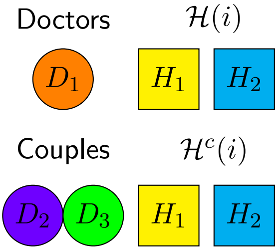

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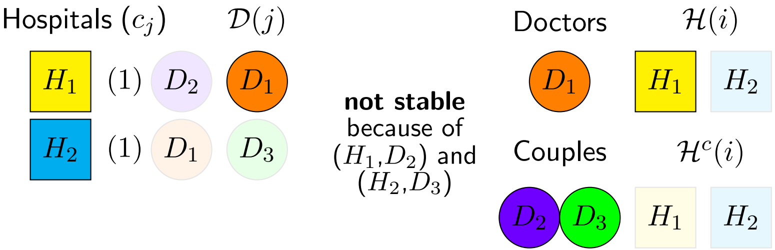

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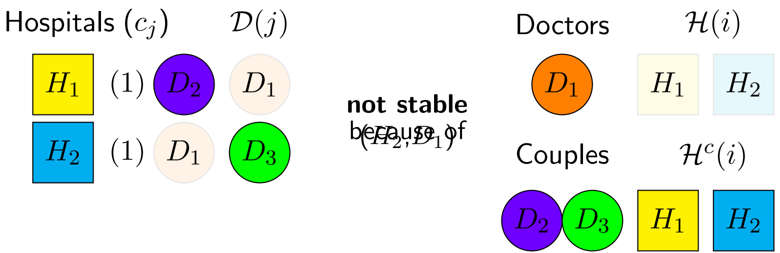

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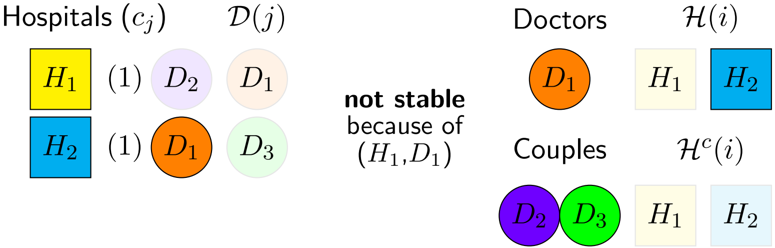

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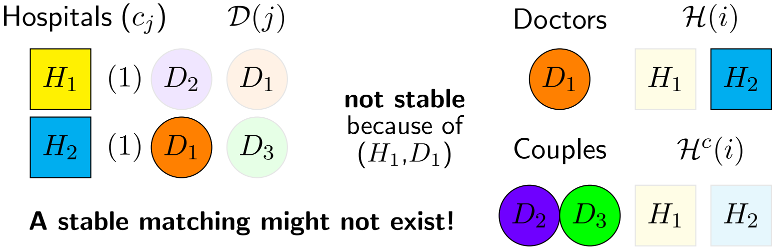

A stable matching might not exist!

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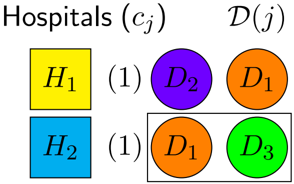

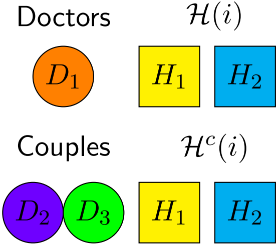

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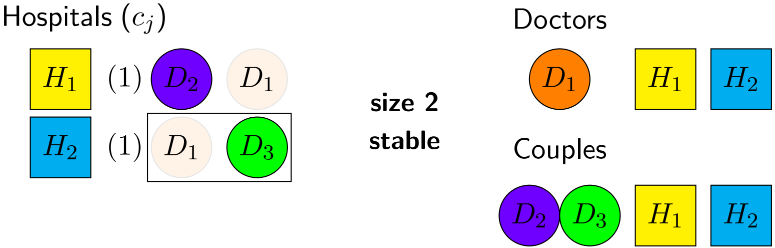

HRCT literature review

3 stability criteria

- MM stability by McDermid and Manlove (2010)
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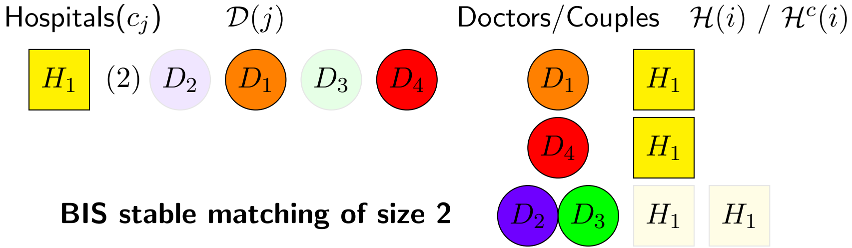

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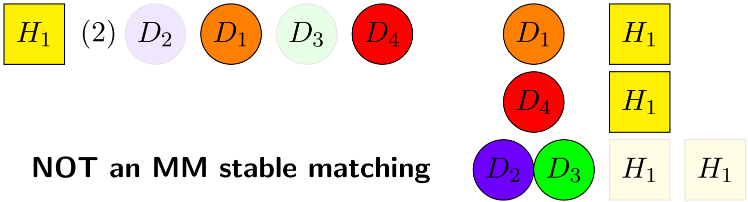
Hospitals(c_j) $\mathcal{D}(j)$ Doctors/Couples $\mathcal{H}(i)$ / $\mathcal{H}^c(i)$



NOT an MM stable matching

Computational experiments – Model improvements

Some **improvements** to the models:

- Some valid inequalities for stability constraints
- Use dummy variables to reduce the number of non-zero elements in the model

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Number of randomly generated instances solved in less than 3600 seconds **for MM stability** using Gurobi 7.5.2

Instance					Literature software		Our model		Our improved model	
n_d	n_c	n_h	$\sum c_j$	inst.	opt	time	opt	time	opt	time
5	0	3	5	10	**10**	0	**10**	0	**10**	0
505	50	50	505	9	**9**	34	**9**	192	**9**	1
1005	100	100	1005	9	**9**	329	6	1624	**9**	1
2505	250	250	2505	9	1	3530	1	3537	**9**	3
5005	500	500	5005	10	0	3600	0	3600	**10**	7