

Programa de Doctorado en Matemáticas

# Actividad Formativa

## ANUNCIO DE CICLO CONFERENCIAS

Geometry of real hypersurfaces in hyperbolic spaces

a cargo de

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The theory of surfaces in the Euclidean space and representation theory of Lie groups are two classical areas in Mathematics. A natural extension of both arises from considering nonflat ambient spaces with large isometry group. Thus, there are very beautiful and profound results in the interplay between submanifold geometry and isometric actions not only in spaces of constant curvature, but also in other contexts, such as compact symmetric spaces or 3-dimensional homogeneous spaces.

The study of submanifolds and isometric actions on hyperbolic spaces over the complex numbers, the quaternions and the octonions frequently requires specific techniques. The main objective of this minicourse is to present some of these ideas and recent results from an introductory viewpoint, with special focus on the geometry of real hypersurfaces (i.e. submanifolds of real codimension one) on these ambient spaces. A powerful approach stems from the fact that hyperbolic spaces are noncompact symmetric spaces. Thus, we will talk about symmetric spaces of noncompact type, their algebraic structure and their Lie group model, and how this underlying structure interacts with the geometry of certain types of hypersurfaces with a high degree of symmetry.

Días: Lunes 18, martes 19 y miércoles 20 de febrero de 2019.
Lugar: Sala de Grados 2, Facultad de Ciencias, Campus de Puerto Real.
Horario: De 15:30h a 18:00h.
Más información: http://c101.uca.es/eventos

Organiza: Pr. José M. Espinar, Investigador Ramón y Cajal, Universidad de Cádiz.

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# Geometry of real hypersurfaces in hyperbolic spaces

### Miguel Domínguez Vázquez Abstract

The theory of surfaces in the Euclidean space and representation theory of Lie groups are two classical areas in Mathematics. A natural extension of both arises from considering nonflat ambient spaces with large isometry group. Thus, there are very beautiful and profound results in the interplay between submanifold geometry and isometric actions not only in spaces of constant curvature, but also in other contexts, such as compact symmetric spaces or 3-dimensional homogeneous spaces.

The study of submanifolds and isometric actions on hyperbolic spaces over the complex numbers, the quaternions and the octonions frequently requires specific techniques. The main objective of this minicourse is to present some of these ideas and recent results from an introductory viewpoint, with special focus on the geometry of real hypersurfaces (i.e. submanifolds of real codimension one) on these ambient spaces. A powerful approach stems from the fact that hyperbolic spaces are noncompact symmetric spaces. Thus, we will talk about symmetric spaces of noncompact type, their algebraic structure and their Lie group model, and how this underlying structure interacts with the geometry of certain types of hypersurfaces with a high degree of symmetry.

#### Contents

- 1. Symmetric spaces of noncompact type: definitions, examples and basic properties.
- 2. The rank one case: root space decomposition of the Lie algebra of the isometry group, Iwasawa decomposition, Lie group model, hyperbolic spaces as Damek-Ricci spaces.
- 3. Homogeneous hypersurfaces, Hopf and curvature-adapted real hypersurfaces, hypersurfaces with constant principal curvatures, isoparametric hypersurfaces: examples and classifications.

#### Main References:

- J. C. Díaz-Ramos, M. Domínguez-Vázquez, V. Sanmartín-López: Submanifold geometry in symmetric spaces of noncompact type, arXiv:1901.04552 [math.DG]. (The first four sections of this survey discuss most of the contents of the minicourse.)
- 2. J. Berndt, S. Console, C. Olmos: Submanifolds and holonomy, Second Edition, CRC Press, Boca Raton, FL, 2016. (It contains comprehensive information on isometric actions and submanifold geometry on symmetric spaces.)
- 3. T. E. Cecil, P. J. Ryan: Geometry of hypersurfaces, Springer Monographs in Mathematics, Springer, New York, 2015. (It includes several chapters on the geometry of hypersurfaces in complex and quaternionic hyperbolic spaces, with focus on Hopf real hypersurfaces.)