“Lion and Man”

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(University of Seville)

(Join work with U. Kohlenbach, B. Piątek and A. Nicolae)

V Jornadas Doctorales
Universidad de Cadiz, Noviembre 2019

Observaciones acerca del desarrollo del trabajo

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Motivación del problema

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Análisis del problema

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1. Gobierno Español: Programa Contratos Juan de la Cierva incorporación. Beneficiaria: Adriana Nicolae. Tutor: Genaro López (2016-2018).

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4. Junta de Andalucía: Grupo FQM 127 (Responsable Tomás Domínguez).
5. Instituto matemático de la Universidad de Sevilla (IMUS): Programa Research au pair (Junio 2018)
6. Instituto Matemático de Oberwolfach (MFO): Programa Research au Pair (Marzo 2019).

Oberwolfach

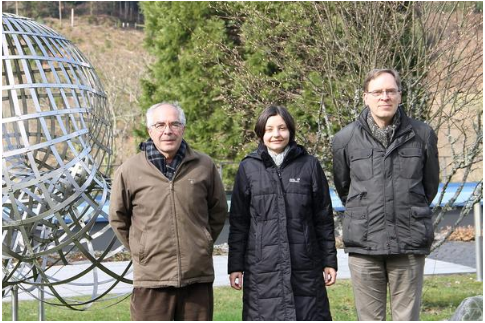

Figure: Research Au Pair in Oberwolfach

Aim of this talk

In these talks we analyze a discrete pursuit-evasion game, based on Rado's lion and man celebrated problem. Pursuit-evasion games go back a long way. Their origin could be placed in the fifth century BC when Zeno of Elea proposed his eternal paradoxes.

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> *A lion and a man move in a closed circular arena have equal maximum speeds. What tactics should the lion employ to be sure of his meal?*

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(Besicovitch): Not true if the man is not confined to move on the boundary of the disc – the man can escape following a polygonal spiraling path.

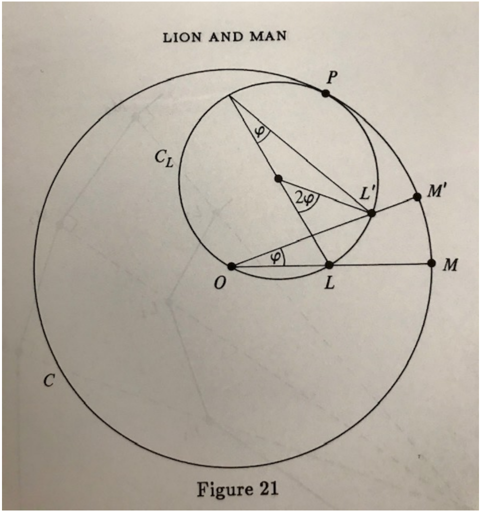
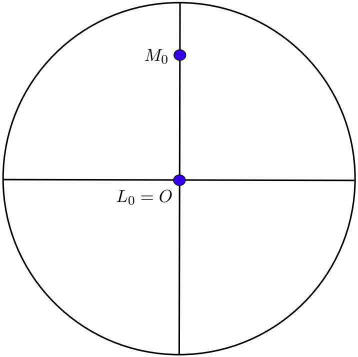


Figure 21

Besicovitch's polygonal spiraling path

Split time into a sequence of intervals $t_1, t_2, t_3 \dots$
At step $i+1$:

- man: $M_i \to M_{i+1}$ running straight for time t_{i+1} s.t. $M_{i+1}M_i \perp OM_i$;
- lion: $L_i \to L_{i+1}$ following "radius rule".



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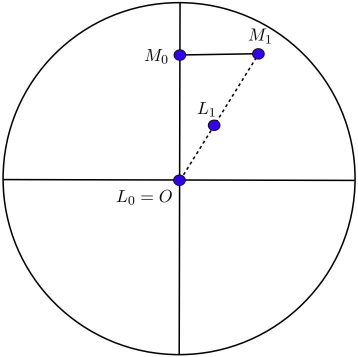

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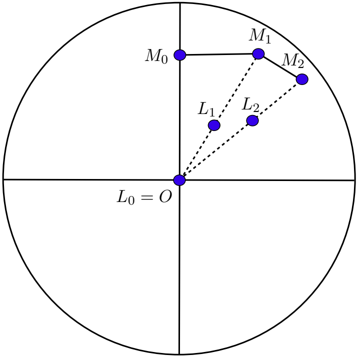

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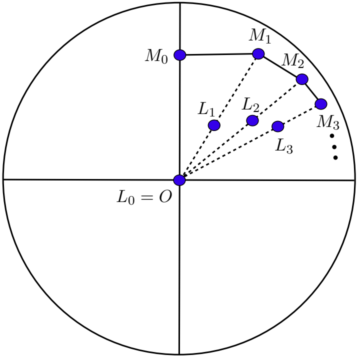

$OM_{i+1}^2 = OM_0^2 + \sum_{n=1}^{i+1} t_n^2 \Rightarrow$ condition: $\sum_{n=1}^{\infty} t_n^2 < R^2 - OM_0^2$, $\sum_{n=1}^{\infty} t_n = \infty$.

Geodesic spaces

(X,d) metric space, $x,y \in X$

Geodesic from x to y: $c:[0,l] \subseteq \mathbb{R} \to X$ with $c(0)=x, c(l)=y$ and

$$d\left(c(t),c(t')\right) = \left|t-t'\right|, \quad \forall t,t' \in [0,l].$$

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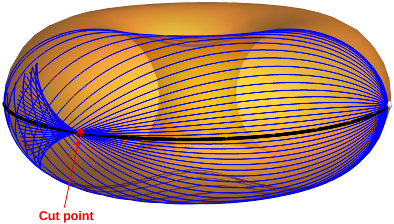

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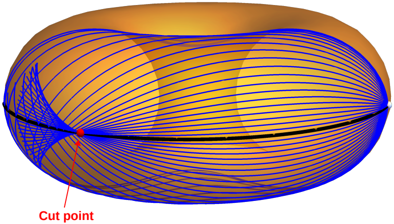

X (uniquely) geodesic space: $\forall x,y \in X$ can be joined by a (unique) geodesic.

Strong convexity

X unique geodesic and $A \subset X$,

- A convex: $\forall x, y \in A$, $[x, y] \subseteq A$.
- A strongly convex: $\forall x, y \in A$, $[x, y] \subseteq A$ and there is no other local geodesic in A from x to y.

I. Chavel, Riemannian geometry–a modern introduction. Cambridge University Press, Cambridge (1993).

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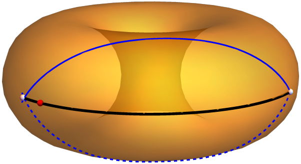

$$CBB\left(\frac{-1}{r(R-r)}\right) \qquad CAT\left(\frac{1}{r(R+r)}\right)$$

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Betweenness property

X uniquely space, $A \subseteq X$ convex

A has the betweenness property (BWP): $\forall x, y, z, w \in A$ parwise distinct,

$$\left.\begin{array}{l} y \in [x,z] \\ z \in [y,w] \end{array}\right\} \Rightarrow y, z \in [x,w].$$

A. Papadopoulos, Metric Spaces, Convexity and Nonpositive Curvature. European Math. Soc., Zürich (2005)

G. López-Acedo, A. Nicolae, B. Piątek, “Lion-Man” and the Fixed Point Property, Geometria Dedicata, 202 (2019), 69-80

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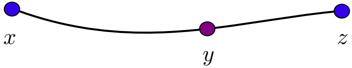

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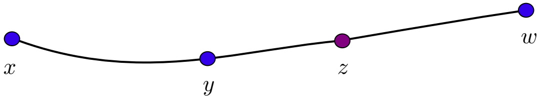

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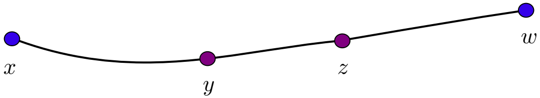

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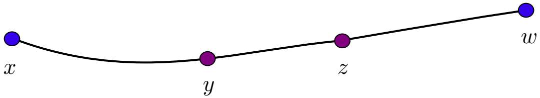

strong convexity of A $\Leftrightarrow$ BWP of A

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Uniform betweennes

We say that X satisfies the uniform betweenness property (UBP) if for all $\varepsilon, a, b > 0$ there exists $\theta > 0$ such that for all $x, y, z, w \in X$ we have

$$\left.\begin{array}{l} \mathrm{sep}\{x,y,z,w\} \geq a \\ \mathrm{diam}\{x,y,z,w\} \leq b \\ \mathsf{dist}(y,[x,z]) < \theta \\ \mathsf{dist}(z,[y,w]) < \theta \end{array}\right\} \quad \Rightarrow \quad \max\{\mathsf{dist}(y,[x,w]), \mathsf{dist}(z,[x,w])\} < \varepsilon.$$

U. Kohlenbach, G. López and A. Nicolae. A quantitatice Analysis of "Lion-Man" game , Oberwolfach Prepint 2019, 10.14760/OWP-2019-18

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A mapping $\Theta : (0,\infty) \times (0,\infty) \times (0,\infty) \to (0,\infty)$ providing for given $\varepsilon, a, b > 0$ such a $\theta = \Theta(\varepsilon, a, b)$ is called a modulus of uniform betweenness.

U. Kohlenbach, G. López and A. Nicolae. A quantitatice Analysis of "Lion-Man" game , Oberwolfach Preprint 2019, 10.14760/OWP-2019-18

Strict convexity

A point $z \in X$ belongs to a geodesic segment with endpoints x and y if and only

$$d(x,y) = d(x,z) + d(z,y),$$

equivalently, there exists $t \in [0,1]$ such that $d(z,x) = td(x,y)$ and $d(z,y) = (1-t)d(x,y)$.

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Let (X,d) be a geodesic space. We say that X is strictly convex if for all $z,x,y \in X$ with $x \neq y$ and all midpoints $m(x,y)$ of x and y we have

$$d(z,m(x,y)) < \max\{d(z,x),d(z,y)\}. \tag{1}$$

- Strictly convex geodesic spaces are uniquely geodesic.
- Any normed vector space is a geodesic space. For this class of spaces, strict convexity is actually equivalent to the existence of unique geodesics between any two points. However, in general this equivalence fails.

Uniform Convexity in geodesic spaces

A geodesic space (X,d) is uniformly convex (UC) if for all $\varepsilon \in (0,2]$ and $r > 0$ there exists $\delta \in (0,1]$ such that for all $z,x,y \in X$ and all midpoints $m(x,y)$ we have

$$\left.\begin{array}{l} d(z,x) \leq r \\ d(z,y) \leq r \\ d(x,y) \geq \varepsilon r \end{array}\right\} \quad \Rightarrow \quad d(z,m(x,y)) \leq (1-\delta)r.$$

A mapping $\eta : (0,2] \times (0,\infty) \to (0,1]$ nonincreasing in the second variable and providing for given $r > 0$ and $\varepsilon \in (0,2]$ such a $\delta = \eta(\varepsilon,r)$ is called a monotone modulus of uniform convexity.

A. Naor, L. Silberman: Poincaré inequalities, embeddings, and wild groups. Compos. Math. **147**, 1546–1572 (2011) .

Relation UC and UUG

Theorem (Kohlenbach, López-Acedo, Nicolae)

Let (X,d) be a uniformly convex geodesic space that admits a monotone modulus of uniform convexity η. The X is uniformly uniquely geodesic and $\Phi : (0,\infty)\times(0,\infty)\to(0,\infty)$ defined by

$$\Phi(\varepsilon,b)=\varepsilon\eta\left(\frac{\varepsilon}{b+\varepsilon},b+\varepsilon\right)$$

is a modulus of uniform uniqueness for X.

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G. López-Acedo, A. Nicolae, B. Piątek, “Lion-Man” and the Fixed Point Property, Geometria Dedicata, 202 (2019), 69-80

U. Kohlenbach, G. López-Acedo and A.Nicolae, A quantitative analysis of the “Lion-Man” game, submitted.

Spaces with the UBWP

- L_p space over a measurable space is p-uniformly convex if $1 < p < +\infty$.

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- L_p space over a measurable space is p-uniformly convex if $1 < p < +\infty$.
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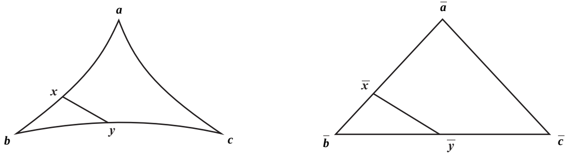

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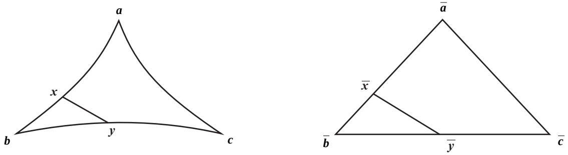

$$d(x,y) \leq d_{M^2_\kappa}(\bar{x},\bar{y})$$

- Alexandrov spaces of curvature bounded below, CBB(κ) ("fat" triangles) + extendable geodesics.

The Lion-Man game in uniquely geodesic spaces

(X, d) geodesic space, $A \subseteq X$ nonempty, convex

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$D > 0$, $L_0, M_0 \in A$ starting points of the lion and the man, resp.
At step $i+1$: $D_i = d(L_i, M_i)$

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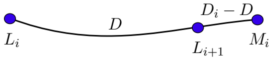

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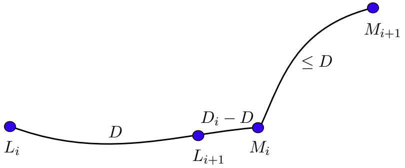
- man: $M_i \rightarrow M_{i+1} \in A$ s.t. $d(M_i, M_{i+1}) \leq D$.

The lion wins: $d(L_{i+1}, M_i) \rightarrow 0$. Otherwise the man wins.



The solution of the Lion-Man game

The lion wins (i.e. $d(L_{i+1}, M_i) \to 0$) iff either of the following holds:

(1) $\exists i_0 \in \mathbb{N}$, $D_{i_0} \leq D$. In this case, $L_{i+1} = M_i$, $\forall i \geq i_0$;

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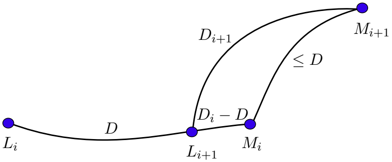
(1) $\exists i_0 \in \mathbb{N}$, $D_{i_0} \leq D$. In this case, $L_{i+1} = M_i$, $\forall i \geq i_0$;

(2) $D_i > D, \forall i \in \mathbb{N}$ and $D_i \to D$.
(D_i) nonincreasing:

$$D_{i+1} \leq d(L_{i+1}, M_i) + d(M_i, M_{i+1}) = D_i - D + d(M_i, M_{i+1}) \leq D_i.$$

The man wins iff $D_i > D, \forall i \in \mathbb{N}$ and $D_i \to D^* > D$.



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Theorem (Kohlenbach, López-Acedo, Nicolae)

Let $A \subseteq X$ be a nonempty, convex and bounded set of diameter $b \geq D$ where the Lion-Man game is played. Take $N \in \mathbb{N}$ such that $b+1 < ND$.

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where

$$\Omega_{D,b,\Theta}(\varepsilon) = N + N \left\lceil \frac{b}{\Theta^N(\varepsilon^*), b)} \right\rceil$$

and

$$0 < \varepsilon^* \leq \min\left\{ \frac{1}{N}, \frac{D}{2}, \frac{\varepsilon}{2} \right\}.$$

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Final Comments

- Non uniquely geodesic spaces.

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- Non uniquely geodesic spaces.
- Characterization of domains where the lion always wins. Existence of geodesic rays.
- Connection with the FPP.

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- C has the fixed point property (FPP) for nonexpansive mappings: $\{x \in C : d(x,Tx) = x\} \neq \emptyset$
- C has the approximate fixed point property (AFPP) for nonexpansive mappings:

$$\forall T : A \to A \text{ nonexpansive},\ \inf\{d(x,Tx) : x \in C\} = 0.$$

The lion and man game and FPP

(X,d) metric space, $b,\lambda \geq 1$, $\varepsilon \geq 0$

The image of $\gamma : [0,\infty) \to X$ (or γ itself) is a:

- geodesic ray: γ is distance-preserving.
- b-directional curve:

$$|s-t| - b \leq d(\gamma(s),\gamma(t)) \leq |s-t|, \quad \forall s,t \geq 0.$$

G. López-Acedo, A. Nicolae, B. Piątek, Geodesic rays, the “Lion-Man” game, and the fixed point property, arXiv:1905.08334 [math.MG].

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Theorem

For any convex and closed subset A of a complete, Gromov hyperbolic CAT(0) space, TFAE:

(i) no geodesic rays in A;

(ii) no directional curves in A;

(iii) A has the AFPP for nonexpansive mappings;

(iv) A has the FPP for nonexpansive mappings;

(v) the lion always wins the lion and man game played in A.

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